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Technical Notes

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Subsonic/Supersonic Yawed Gust over an Airfoil

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Introduction

IN this Note the fundamental problem of a two-dimensional flat plate immersed in a convected, yawed sinusoidal gust is considered for both subsonic and supersonic mean flow. Graham[1] proposed for the incompressible case an integral equation derived from Possio's formulation of two-dimensional unsteady compressible flow. His method of solution involves a Chebyshev expansion of part of the kernel function and can be considered exact because, as Graham remarks, these series are considered highly convergent. The case of compressible subsonic mean flow was also solved by Graham,[2] who presents similarity rules that reduce the general problem in two classes: one equivalent to the incompressible yawed gust (subcritical case) and another equivalent to the parallel gust in compressible flow (supercritical case). Among other numerical solutions, the work of Nagashima and Tanida[3] deserves to be mentioned, where Possio's integral equation is solved by the finite element method. However, this is an acceleration potential formulation, and its kernel function is therefore more complicated than the kernel used in the velocity potential formulation adopted in the present work.

Several analytical approximations have also been proposed, as the first-order solutions of Osborne[4] and Amiet.[5] Second-order solutions were also provided by Graham and Kullar[6] and Amiet.[7] An interesting approximate analytical solution was proposed by Martinez and Widnall[8] for the case of an oblique gust of short wavelength, where it was considered that, for an infinite span wing, the leading- and trailing-edge responses to a short-wavelength gust are essentially independent. Another generalization for the problem treated here was proposed by Adamczyk,[9] who includes the effect of sweep. This approximate solution was obtained in terms of an infinite series expansion of Mathieu functions.

In the present work, the generalized vortex lattice method[10] is applied to the yawed sinusoidal gust in subsonic flow over a thin airfoil. A new treatment is proposed for the wake to avoid its discretization, and this makes the present method as interesting as the acceleration potential formulation. As mentioned before, the subsonic case can be divided into two classes, namely, the subcritical and supercritical cases. Application of the generalized vortex lattice method is straightforward in the supercritical case, whereas in the subcritical case a new elementary solution is presented. Finally, following a suggestion by Graham,[1] an extension of the study of the yawed gust problem for a supersonic mean flow is also discussed.

Mathematical Flow Model

Consider a reference frame fixed to the airfoil, as depicted in Fig. 1, with origin at the midchord position. If one follows the work of Graham,[2] for example, the differential equation governing the velocity potential Φ due to a small-amplitude vertical gust and the associated boundary condition can be written as

$$\Phi_{XX} \pm \Phi_{ZZ} + \chi^2 \Phi = 0 \tag{1}$$

$$\Phi_Z = \mp (L W_0/\beta)\, e^{\mp ikX/\beta^2} \tag{2}$$

The upper and lower signs represent subsonic or supersonic flows, respectively. Here W_0 is the amplitude of the vertical gust, $2\pi/\lambda$ and $2\pi/\mu$ are the streamwise and spanwise wavelengths, $\omega = \lambda U$ is the angular frequency of the oscillation, M is the undisturbed flow Mach number, U is the freestream velocity, $2L$ is the airfoil chord, and the other nondimensional variables are

$$X = x/L, \qquad Z = \beta(z/L), \qquad \mu = \nu/L$$

$$k = \omega L/U, \qquad K = kM/\beta^2$$

$$\beta^2 = \pm(1 - M^2), \qquad \chi = \left\{K^2 - [\nu^2/(1 - M^2)]\right\}^{\frac{1}{2}}$$

The pressure coefficient C_p derived from the linearized pressure equation reads

$$C_p = -(2/LU)\, e^{\pm iKMX}[\Phi_X \pm i(K/M)\Phi] \tag{3}$$

The boundary condition on the wake, for subsonic flow, is

$$\delta C_p = 0 = i(K/M)\delta\Phi + (\delta\Phi)_X \tag{4}$$

It is important to emphasize that the conditions at infinity for Eqs. (1) are identically satisfied through the introduction of the elementary solutions defined for each equation. In the subsonic case, for example, it is required that disturbances die away at infinity. The integral equation that relates the velocity component normal to the airfoil to the velocity at an arbitrary field point reads[11]

$$\Phi_Z(X_P) = \frac{1}{4}\int_{-1}^{\infty}[D_0(\chi R)]_{ZZ}\,\delta\Phi\,\mathrm{d}X \tag{5}$$

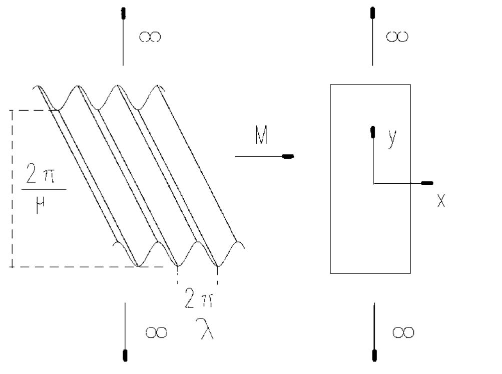

Fig. 1 Airfoil reference frame for oblique sinusoidal gust encounter.

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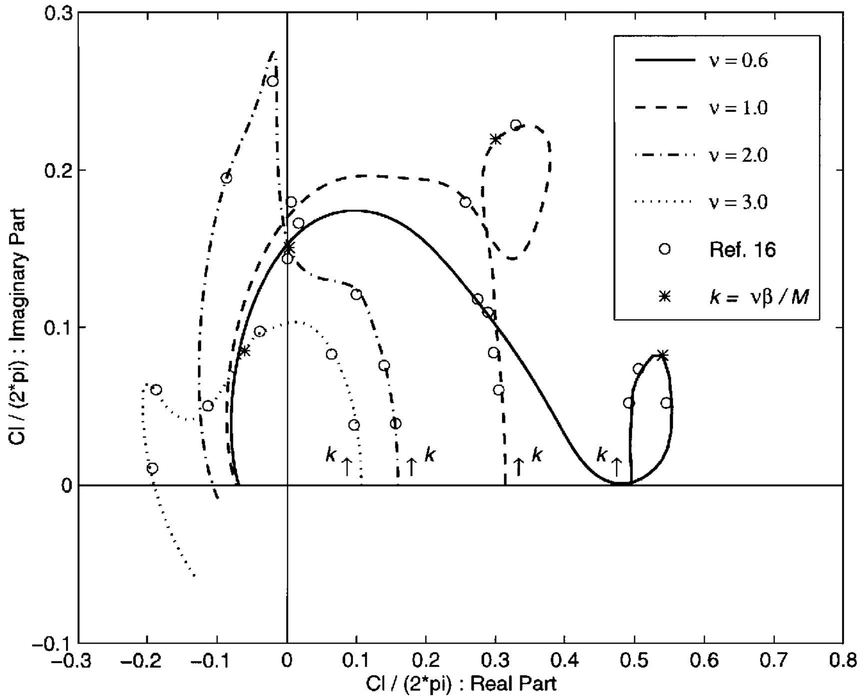
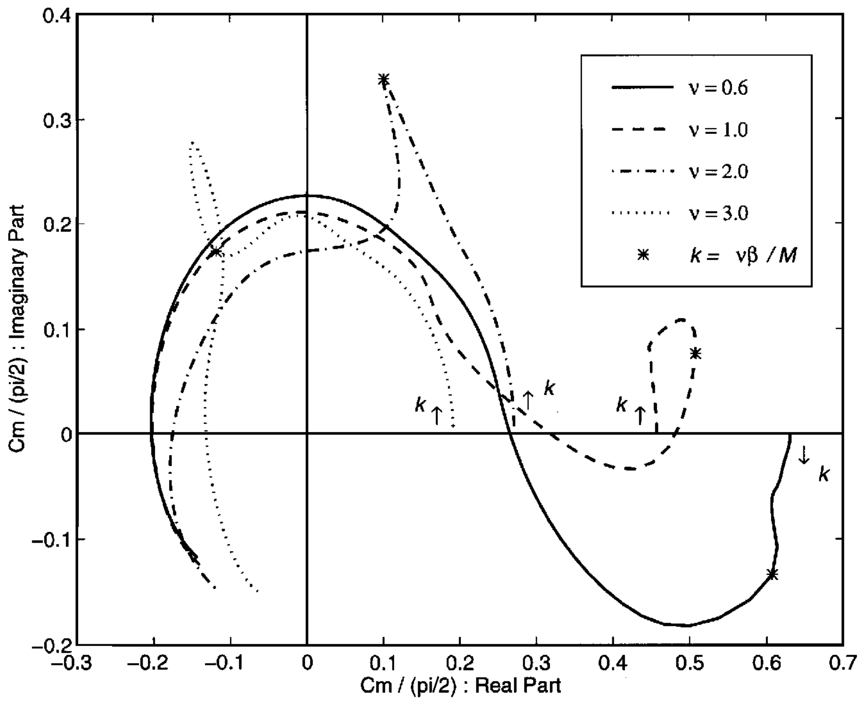


a) Lift coefficient



b) Pitching-moment coefficient

Fig. 2 Phase diagrams for the case of subsonic flow ($M = 0.8$) and oblique gusts ($\nu > 0$).

where $R = [(X_P - X)^2 + Z^2]^{1/2}$ and X_P is the field-point coordinate. The corresponding elementary solutions of Eq. (5) can be written as follows.

Real χ, supercritical case:

$$D_0(\chi R) = -Y_0(\chi R) - iJ_0(\chi R) = -iH_0^{(2)}(\chi R) \qquad (6)$$

Imaginary χ, subcritical case:

$$D_0(\chi R) = (2/\pi)K_0(|\chi|R) \qquad (7)$$

Expression (7) has been derived from Eq. (6), assuming imaginary arguments, and application of the boundary condition at infinity.

The yawed sinusoidal gust incident on a thin airfoil in undisturbed supersonic flow is simpler to solve than the subsonic one, and lift and moment coefficients can be obtained from Refs. 12 and 13. However, the change in the tabulated f_n functions used in the parallel gust case is not straightforward, and they now should read

$$f_n(M, \chi, K) = \int_0^1 X^n e^{-2iKMX} J_0(2\chi X)\,\mathrm{d}X \qquad (8)$$

Given the differential equations (1) and the boundary conditions (2), one can find Φ distributions that are similar to each other if the parameters are varied in an adequate way. This study was done by Graham[2] for subsonic flows, and it is possible to show that the same relations valid for the supercritical subsonic case still hold in the supersonic case.

Numerical Solution

In the present section, only the numerical solution of the subsonic case is considered. The solution is established by solving Eq. (5) in the limit $Z \longrightarrow 0$, subjected to boundary conditions (2) and (4) for the profile and wake, respectively. The profile is assumed to be divided into NX panels, such that $X_n = -1 + n(2/NX)$, $n = 0, 1, \ldots, NX$. Note that $X_0 = -1$ corresponds to the leading edge, whereas the trailing edge is given by $X_{NX} = 1$.

For the nth panel, discretization of Eq. (5) leads to

$$A_{mn} = \frac{1}{4}\int_{X_{n-1}}^{X_n} \lim_{Z \longrightarrow 0}\{[D_0(\chi R)]_{ZZ}\}\,\mathrm{d}X \tag{9}$$

where A_{mn} denotes the contribution of the nth panel to the velocity at point $X_P = 0.5(X_m - X_{m-1})$. When X_P does not belong to the nth panel, Eq. (9) in the supercritical case becomes

$$A_{mn} = \frac{\chi}{4}\int_{X_{n-1}}^{X_n} \frac{Y_1(\chi R) + iJ_1(\chi R)}{R}\,\mathrm{d}X \tag{10}$$

or

$$A_{mn} = \mathrm{sgn}(X_n - X_P)\frac{\chi}{4}$$

$$\times\left\{\left[\int_{\bar{R}_{n-1}}^{\bar{R}_n} Y_0(\bar{R})\,\mathrm{d}\bar{R} - Y_1(\bar{R}_n) + Y_1(\bar{R}_{n-1})\right]\right. \tag{11}$$

$$\left. + i\left[\int_{\bar{R}_{n-1}}^{\bar{R}_n} J_0(\bar{R})\,\mathrm{d}\bar{R} - J_1(\bar{R}_2) + J_1(\bar{R}_1)\right]\right\}$$

Here $\bar{R} = |\chi|R = |\chi||X - X_P|$, whereas sgn( ) denotes the sign of the argument. One should note that the Bessel functions' integrals may be written as rational expansions,[14] such that the choice of expression (10) or (11) can lead to CPU times that differ at least one order of magnitude if the code is written in a vectorized environment.

When X_P does belong to the nth panel, the definition of the normal velocity is obtained at the limit as given in Ref. 15. Therefore, for the supercritical case one has

$$A_{nn} = 2A_{n-\frac{1}{2}+\epsilon,n} + (1/\pi\epsilon) + i(\chi^2/4)\epsilon \tag{12}$$

where $A_{n-(1/2)+\epsilon,n}$ denotes the integral in expression (10), for example, from $0.5(X_n - X_{n-1}) + \epsilon$ to X_n. Numerically speaking, ϵ should be no larger than $\frac{1}{50}$th of the panel length.

Similar expressions can be derived for the subcritical case, written in the form

$$A_{mn} = -\mathrm{sgn}(X_n - X_P)\frac{|\chi|}{2\pi}$$

$$\times\left[\int_{\bar{R}_{n-1}}^{\bar{R}_n} K_0(\bar{R})\,\mathrm{d}\bar{R} + K_1(\bar{R}_n) - K_1(\bar{R}_{n-1})\right] \tag{13}$$

$$A_{nn} = 2A_{n-\frac{1}{2}+\epsilon,n} + (1/\pi\epsilon) \tag{14}$$

Regarding the wake, it is possible to write its influence in terms of the value of $\delta\Phi$ at the trailing edge, i.e., $\delta\Phi_{NX+1}$. To do this, one must first recognize that expression (4) is identically satisfied if $\delta\Phi = \delta\Phi_{NX+1}e^{i(K/M)(1-X)}$. Therefore, Eq. (5) on the wake yields

$$A_{m,NX+1} = \frac{1}{4}\int_{1}^{\infty} e^{i(K/M)(1-X)} \lim_{Z \longrightarrow 0}\{[D_0(\chi R)]_{ZZ}\}\,\mathrm{d}X \tag{15}$$

The preceding integral may be evaluated numerically from 1 to a large number (1000K, for example). When this is done, one is left with a system of NX equations in $NX + 1$ unknowns. The closure of the system of equations may be achieved by enforcing that C_p is zero at the trailing edge.

Results

Figure 2 shows lift and moment (about the semichord point) coefficients for several gust angles and a high subsonic Mach number value ($M = 0.8$). All numerical results have been obtained for a discretization of 200 panels along the profile chord, and the order of the error is NX^{-1}. The overall agreement with results presented by Atassi[16] is very good. One can see that the behavior of the curves departs from the classical parallel and incompressible oblique cases and deserves more studies regarding combined effects of obliqueness and compressibility. The presence of knots on the phase diagrams can also be observed in some of the results of Nagashima and Tanida.[3] Finally, for each ν curve there is a switch from the subcritical solution to the supercritical one as the reduced frequency increases, with the border point given by $k = \nu\beta/M$.

Conclusions

It has been shown that the present formulation is capable of handling the problem of a thin airfoil immersed in a yawed sinusoidal gust, both in subsonic and supersonic flow. In subsonic regime the wake is directly treated by numerical integration, with no need of modeling it by panels. From a numerical point of view, the kernels presented are notably simpler than the ones employed in Possio's formulation for both incompressible yawed and compressible cases. Finally, in supersonic regime the expression for the integrals f_n has been generalized to treat yawed gusts.

Acknowledgments

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